English

The Niels Henrik Abel mathematics competition 2012–2013

First round 8. November 2012

Do not turn the page until told to by your teacher!

The first round of the Abel competition consists of 20 multiple choice problems to be solved in 100 minutes. Only one of the five alternatives is correct. Write your answers in the lower left hand side of the form.

You get 5 points for each correct answer, 1 point for a blank answer, and 0 points for a wrong answer. This yields a total between 0 and 100 points. A totally blank response results in 20 points.

No aids other than scratch paper and writing implements are allowed.

Please note: Although efforts have been made to make this translation correct, only the Norwegian versions are official, and your answers are marked according to these.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name	Date of birth
Address	Gender F ☐ M ☐
Post code	Post office
School	Class

Answers

1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

For the teacher

Correct: ☐ $\cdot 5 =$ ☐

Unanswered: $+$ ☐

Points: $=$ ☐

Problem 1

What is $3^6 \cdot 9^{12}$ equal to?

A 12^{18} **B** 9^{15} **C** 15^{18} **D** 3^{18} **E** None of these.

Problem 2

Per, Ragnar, and Lars live in the same neighbourhood. They have found out that the straight line distance from Per to Ragnar is 250 m, and from Ragnar til Lars, it is 300 m. What is the best one can say about the distance from Per to Lars based on this information?

A The distance is precisely 550 m.
B The distance can be anything between 0 m and 550 m.
C The distance can be anything between 50 m and 550 m.
D The distance can be anything between 250 m and 300 m.
E The distance can be anything at all.

Problem 3

How many three digit numbers are such that their first digit equals the sum of the last two digits?

A 45 **B** 48 **C** 50 **D** 54 **E** 55

Problem 4

How many different prime factors does 360 have?

A 2 **B** 3 **C** 4 **D** 5 **E** 6

Problem 5

Lars is twice as old as Kari, and Kari's age is one third the age of Stian. Five years ago, Lars was half as old as Stian was. What is the sum of the present ages of Kari, Lars, and Stian?

A 30 years **B** 54 years **C** 60 years **D** 90 years **E** 120 years

Problem 6

How many positive integers n exist, such that $784/n$ is an integer?

A 7 **B** 8 **C** 14 **D** 15 **E** 20

Problem 7

ABC is an equilateral triangle. A circle with radius 1 is tangent to the line AB at the point B and the line AC at the point C. What is the side length of ABC?

A $\frac{\sqrt{3}}{2}+1$ **B** $\sqrt{3}$ **C** $\frac{\sqrt{3}}{2}$ **D** $\frac{2\sqrt{3}}{3}$ **E** 2

Problem 8

You throw three ordinary six-sided dice. What is the probability that you get one odd number and two even numbers?

A $1/4$ **B** $3/8$ **C** $4/27$ **D** $1/2$ **E** $1/3$

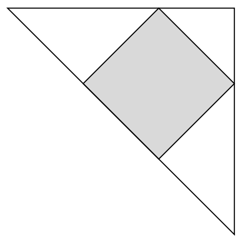
Problem 9

Which number is the largest?

A $0{,}3$ **B** $\sqrt{0{,}095}$ **C** $0{,}1/0{,}30$ **D** $0{,}5^2$ **E** $240/723$

Problem 10

A square is placed within an isosceles right triangle whose shorter sides have length 1, as shown in the figure. What is the area of the square?



A $2/9$ **B** $2/10$ **C** $\sqrt{3}/9$ **D** $3/10$ **E** $\sqrt{2}/5$

Problem 11

What is the average of all positive three digit numbers?

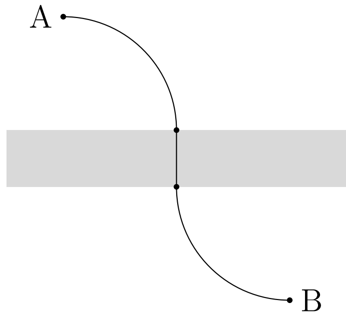
A 500 **B** $549{,}5$ **C** 599 **D** $599{,}5$ **E** 600

Problem 12

The sum of three consecutive integers is a prime p. What is p?

A 2 **B** 3 **C** 11 **D** 13 **E** Impossible to decide.

Problem 13



Karl Erik is riding his bike home from school today. He rides from A to B as shown in the figure. First he rides on a bike path shaped as a quarter circle, then he rides across a 20 m long bridge, and finally he completes the rest of the bike path, which is also shaped as a quarter circle. He rides the same distance before as after the bridge. If the straight line distance from A to B is 100 m, how many metres does Karl Erik ride?

A $40\pi + 20$ **B** $30\pi + 20$ **C** 210 **D** $60\pi + 20$ **E** 40π

Problem 14

Nils's sock drawer contains 9 white, 20 blue, and k black socks. The probability to end up with two black socks if he takes two socks at random from the drawer is $1/30$. What is the value of k?

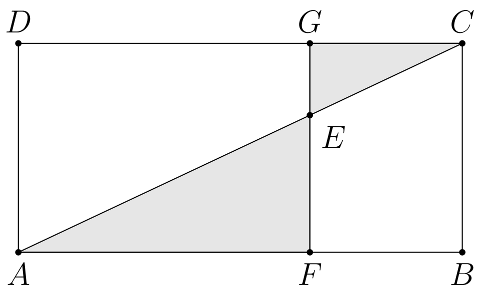
A 5 **B** 6 **C** 7 **D** 8 **E** 9

Problem 15

If $m = 2 \cdot 3 \cdot 4 \cdot 5 \cdot \cdots \cdot 31 \cdot 32$, which statement about m is true?

A $m < 2^{40}$ **B** $2^{40} < m < 2^{70}$ **C** $2^{70} < m < 2^{100}$
D $2^{100} < m < 2^{130}$ **E** $2^{130} < m$

Problem 16



The area of the rectangle $ABCD$ is 1. Assume that E lies on the diagonal AC, and that the line through E parallel to AD and BC meets AB in F and CD in G. La $x = AE/EC$. What is the sum of the areas of AFE and ECG?

A $\dfrac{1+x^2}{2(1+x)^2}$ **B** $\dfrac{2(1+x^2)}{(1+x)^2}$ **C** $\dfrac{2(1+x)^2}{1+x^2}$ **D** $\dfrac{(1+x)^2}{2(1+x^2)}$

E Impossible to determine given x.

Problem 17

What is the sum of the digits of the largest integer $n \leq 2013$ such that the sum of the digits of n equals the product of the digits of n?

A 2 **B** 3 **C** 4 **D** 6 **E** 8

Problem 18

In a triangle ABC the sides satisfy $AB = 5$, $BC = 4$, and $CA > 3$. The area of the triangle is 6. What is the length of the side CA?

A $\sqrt{77}$ **B** $\dfrac{\sqrt{1901}}{5}$ **C** $\dfrac{\sqrt{2012}}{5}$ **D** $\sqrt{73}$ **E** $\dfrac{3\sqrt{197}}{5}$

Problem 19

What is the final digit of the number $1^1+2^2+4^4+503^{503}+1006^{1006}+2012^{2012}$?

A 0 **B** 2 **C** 4 **D** 6 **E** 8

Problem 20

How many real solutions does the equation $x + x^2 + \cdots + x^{2012} = 0$ have?

A 1 **B** 2 **C** 1006 **D** 2011 **E** 2012

The solutions are published on 9 November at 17.00 on

abelkonkurransen.no