English

The Niels Henrik Abel mathematics competition 2013–2014

First round 7. November 2013

Do not turn the page until told to by your teacher!

The first round of the Abel competition consists of 20 multiple choice problems to be solved in 100 minutes. Only one of the five alternatives is correct. Write your answers in the lower left hand side of the form.

You get 5 points for each correct answer, 1 point for a blank answer, and 0 points for a wrong answer. This yields a total between 0 and 100 points. A totally blank response results in 20 points.

No aids other than scratch paper and writing implements are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name	Date of birth
Address	Gender F ☐ M ☐
Post code	Post office
School	Class
Have you participated in the Abel competition before? If so, what year(s)?	

Answers

1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

For the teacher

Correct:	☐ $\cdot\, 5 =$	☐
Unanswered:	$+$	☐
Points:	$=$	☐

Problem 1

How many positive integers divide 2^{10}?

A 2 **B** 9 **C** 10 **D** 11 **E** 512

Problem 2

Two years ago, Petter was three times as old as Ulrik. In two years, Ulrik will be only half as old as Petter. What is the sum of Ulrik's and Petter's current ages?

A 18 **B** 20 **C** 22 **D** 24 **E** 26

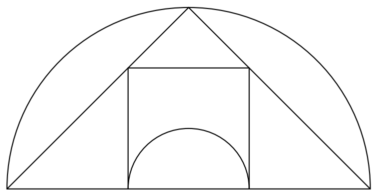
Problem 3

Three real numbers x, y, and z are such that $3x + y = 1$, $3y + z = \frac{1}{2}$, and $3z + x = -\frac{1}{2}$. What is the value of $x + y + z$?

A 1 **B** $\frac{1}{2}$ **C** $\frac{1}{3}$ **D** $\frac{1}{4}$ **E** 0

Problem 4

In the figure, a semicircle is inscribed in a square, which is inscribed in an isosceles triangle, which is inscribed in a semicircle. What is the ratio between the area of the large semicircle and the area of the small semicircle?



A 9 **B** $6\sqrt{2}$ **C** $5\sqrt{3}$ **D** 10
E It depends on the radius of the small semicircle.

Problem 5

Seven houses are situated in a row along a street. Three of the houses are red, three of them are blue, and one house is white. Which of the following statements is correct:

A There must be two red houses next to each other.
B There must be a blue house with a red house next to it.
C If the white house is not next to a blue house, there must be two blue houses next to each other.
D If the white house is painted blue, then two blue houses will be next to each other afterwards.
E None of the previous statements are correct.

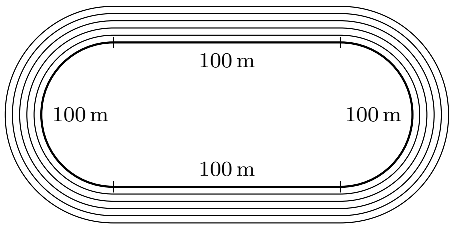
Problem 6

Hadia, Jens, and Siv are collecting cars. You have been told that none of them has more than five cars, and that no two of them have more than seven cars in total. What is the largest number of cars all three can have in total?

A 8 **B** 9 **C** 10 **D** 11 **E** 12

Problem 7

A round at the innermost running track in an athletic field (track one) consists of two straights of 100 m each, and two semicircles of 100 m each. If we assume that each track is 1 m wide, how much longer is a round at track five?



A $8\,\mathrm{m}$ **B** $(4\pi + 8)\,\mathrm{m}$ **C** $4\pi\,\mathrm{m}$ **D** $8\pi\,\mathrm{m}$ **E** $10\pi\,\mathrm{m}$

Problem 8

All numbers can be written in base three, in a way similar to base ten, using one or more successive digits. The difference is that in base three, only the digits 0, 1, and 2 can be used. The numbers which we write in base ten as 1, 2, 3, 4, 5, and so on, are written in base three as 1, 2, 10, 11, 12, and so forth. What number in base ten corresponds to the number 1021 in base three?

A 16 **B** 31 **C** 34 **D** 40 **E** 51

Problem 9

Which of the alternatives equals $\dfrac{1+\sqrt{2}}{\sqrt{2}-1}$?

A $1+\sqrt{2}$ **B** $3+2\sqrt{2}$ **C** $3\sqrt{2}$ **D** $2+\sqrt{2}$ **E** $1+\frac{2}{3}\sqrt{2}$

Problem 10

Gro is to make a six digit PIN code, but she can only use the digits 1, 2, and 3. How many codes are possible, if each digit is to be used at least once?

A 534 **B** 537 **C** 540 **D** 726 **E** 729

Problem 11

ABC is an equilateral triangle. A circle of radius 1 is tangent to the line AB at B and to the line AC at C. What is the side length of ABC?

A $\sqrt{2}$ **B** $\dfrac{2}{\sqrt{3}}$ **C** $\dfrac{4}{3}$ **D** 1 **E** $\sqrt{3}$

Problem 12

Which number is the largest?

A $3.13 \cdot 3.15$ **B** 9.85 **C** $\sqrt{9.61}\pi$ **D** π^2 **E** $\dfrac{\pi^3}{3.15}$

Problem 13

What is the sum of the last two digits of the smallest number which is divisible by both $1 + 2 + 3 + \cdots + 10$ and $1 \cdot 2 \cdot 3 \cdots 10$?

A 0 **B** 1 **C** 2 **D** 5 **E** 9

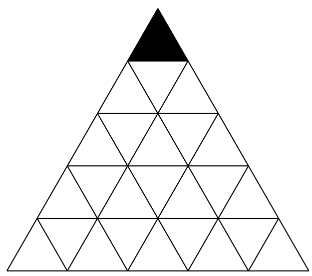
Problem 14

What is the side length of the largest cube which can fit inside a sphere of radius 1?

A $\sqrt{2}$ **B** $\dfrac{2}{\sqrt{3}}$ **C** $\dfrac{4}{3}$ **D** 1 **E** $\sqrt{3}$

Problem 15

In the figure one can move from any cell to a neighbour cell on the row below. Two cells are considered neighbours if they have a side or a corner in common. Starting in the black cell, how many possible paths are there in total to the bottom row? (For example, there are three paths from the black cell to the row below it.)



A 81 **B** 153 **C** 215 **D** 375 **E** 945

Problem 16

What is the the smallest integer n greater than 1 such that the last digit of a^n is the same as the last digit of a, for all possible positive integers a?

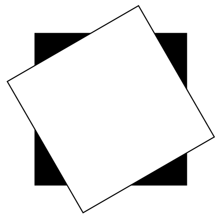
A 4 **B** 5 **C** 9 **D** 10 **E** There is no such integer n.

Problem 17

How many real solutions does the equation $2x^6 + 3x^4 - 2x^2 = 0$ have?

A 1 **B** 2 **C** 3 **D** 5 **E** 6

Problem 18



The figure shows a black square covered by a white square with the same centre, but rotated 30 degrees. Both squares have side length 2. What is the total area of the visible black areas?

A $\frac{8}{\sqrt{3}} - 4$ **B** $\frac{8\sqrt{3}}{25}$ **C** $\frac{16}{\sqrt{3}} - 8$ **D** $\frac{16\sqrt{3}}{25}$ **E** $3\sqrt{3} - 4$

Problem 19

Trine wants to paint a collection of cubes so that each side is one solid colour, and adjacent sides are never painted the same colour. She has only red, blue, green, and yellow paint. Consider two cubes to be painted the same way if they can be turned so that sides facing the same way are the same colour. How many cubes can she paint differently from each other?

A 10 **B** 14 **C** 16 **D** 20 **E** 30

Problem 20

How many perfect squares are there among the numbers 2013, 2020, 2027, . . . , 3595, 3602?

A 0 **B** 1 **C** 2 **D** 3 **E** 4

The solutions are published on 8 November at 17.00 on

abelkonkurransen.no