ABEL KONKURRANSEN

English

The Niels Henrik Abel mathematics competition 2014–2015

Second round   15 January 2015

Do not turn the page until told to by your teacher! The second round of the Abel competition consists of 10 problems to be solved in 100 minutes. The solutions are integers between 0 and 999, inclusive. Write your answers in the lower left hand side of the form.

You get 10 points for each correct answer and 0 points for a blank or wrong answer. This yields a total between 0 and 100 points.

No aids other than scratch paper and writing implements are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name		Date of birth
Address		Gender F ☐ M ☐
Post code	Post office	
School		Class
Citizenship	Email	Mobile phone

Answers

1		6	
2		7	
3		8	
4		9	
5		10	

For the teacher

Correct: ☐ $\cdot 10 =$ ☐

Problem 1

How many six-digit positive integers can you write, if each number must have strictly increasing digits from left to right?

Problem 2

If $a = 13 + \frac{1}{b}$ and $a^2 = 143 + \frac{1}{b^2}$, what is $a + \frac{1}{b}$?

Problem 3

Points A and B have coordinates: $A = (720, 1440)$ and $B = (4, 2)$. The line segment AB intersects the line $x = y$ at the point P. What is the ratio AP/PB?

Problem 4

If $a_0 = 0$, $a_1 = 1$, and $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$, what is the final digit of a_{2014}?

Problem 5

In how many ways can 9 black and 9 white rooks be placed on a 6×6 chess board, so that no white rook can capture a black one?

A rook can capture another piece if it is in the same rank (row) or the same file (column) as the other piece, with no other pieces between the two.

Problem 6

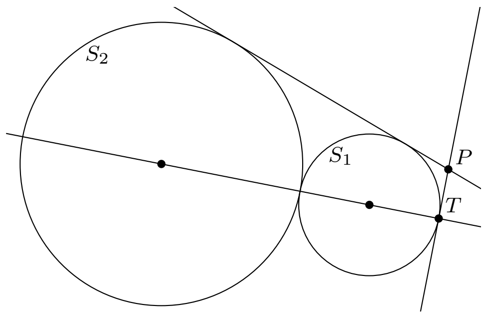
Let A_0 be the set $\{1, 2, 3, 4\}$. Let A_{i+1} be the set of all possible sums which can be obtained by adding two numbers in A_i, where the two numbers do not have to be different. How many unique numbers does A_8 contain?

Problem 7

What is the largest possible value of $426k - 90k^2$ where k must be an integer?

Problem 8

Two circles, S_1 with radius 30 and S_2 with radius 60, are tangent to each other, with each circle exterior to the other. Point T lies on S_1, remote from S_2, where the line through both centres meets S_1. P is a point where the tangent to S_1 at T meets a line which is tangent to both circles. What is the square of the distance between P and T?



Problem 9

A box contains fewer than 1000 pieces of chocolate. Nils wants to divide them into piles of equal size. First, he tries with 15 piles, but 12 pieces are left over. He eats these, and then tries to divide the rest into 16 equal piles. But now there are 13 pieces left over, and he decides to eat these as well. In his third and last attempt, now with 18 equal piles, he gets 14 pieces left over, so he eats these too. In his frustration, he continues eating. But after 19 more pieces, he becomes dizzy and must lie down – and he believes he sees a goose with a small Swede on its back come and steal the remaining chocolate. How many pieces of chocolate were in the box originally?

Problem 10

Four different positive integers a, b, c, and d are such that $a^2 + b^2 = c^2 + d^2$. What is the smallest possible value of $abcd$?

Solutions are posted on 16 January at 17.00 on

abelkonkurransen.no