English

The Niels Henrik Abel mathematics competition 2015–2016

Second round 14 January 2016

Do not turn the page until told to by your teacher! The second round of the Abel competition consists of 10 problems to be solved in 100 minutes. The solutions are integers between 0 and 999, inclusive. Write your answers in the lower left hand side of the form.

You get 10 points for each correct answer and 0 points for a blank or wrong answer. This yields a total between 0 and 100 points.

No aids other than scratch paper and writing implements (including compass and ruler) are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name			Date of birth
Address			Gender F ☐ M ☐
Post code	Post office		
School			Class
Citizenship	Email		Mobile phone

Results list: Note that regardless, we only publish results for the best third

☐ Check the box to allow us to put your name on the results list

Answers

1		6	
2		7	
3		8	
4		9	
5		10	

For the teacher

Correct: ☐ $\cdot 10 =$ ☐

Problem 1

If a and b are positive integers and $a^3 + a^2b - ab^2 - b^3 = 2^{10}$, what is the value of a?

Problem 2

In a right triangle ABC, $\angle BAC = 90°$. The circle having AB as diameter has area 283, and the circle having AC as diameter has area 282. What is the area of the circumcircle of ABC? (The circumcircle of ABC is the circle passing through the points A, B, and C.)

Problem 3

Among the positive integers less than 32, A numbers have exactly four unique positive divisors, B numbers have exactly three unique positive divisors, and C numbers have exactly two unique positive divisors. (A *divisor* is an integer which divides a given integer n with no remainder. Note that 1 and n are counted as divisors of any positive integer n.) What is the product ABC?

Problem 4

Five points A, B, C, D, and E are located in the given order on a circle, so that $AB = BC = CD = DE$ and $\angle ADE = 120°$. What is $\angle CDE$ measured in degrees?

Problem 5

Two positive numbers x and y satisfy $2x - x^2 + 2y - y^2 \geq 2xy + 1$ and $y^2 - x^2 = \frac{1}{3}$. What is y/x?

Problem 6

What is the number of positive integer solutions (a, b) to $2016 + a^2 = b^2$?

Problem 7

Which integer is closest to $\dfrac{444}{\sqrt{111 \cdot 112} - 111}$?

Problem 8

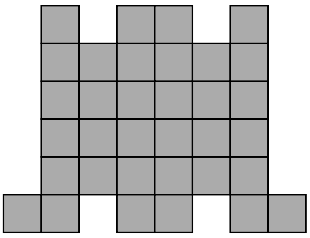
An athletic flea is training by jumping back and forth on the number line. Each training session starts at 0, and consists of one jump each of length 2, 4, 8, 16, 32, 64, 128, and 256, always in that order. In order to have some variety, the flea can freely choose the direction, to the right or to the left, for each jump. In how many different integer locations can the flea end up after a training session?

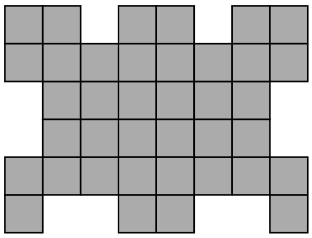
Problem 9

Points A, B, C, and D are located on a circle so that the line segments AC and BD cross each other. The positive integers a and b satisfy $AB = b$, $AD = a$, $BC = \frac{1}{2}a$, and $CD = 2b$. Assuming $\angle BAD = 90°$ and the area of $ABCD$ is less than 1000, what is the largest possible area of $ABCD$?

Problem 10

The figure on the left can be covered by 17 dominoes in M different ways, while the figure on the right can be covered by 19 dominoes in N different ways. (A domino is a 2×1 rectangle. The dominoes must cover each figure entirely with no overlap.) Either M/N or N/M is an integer. What is that integer?





Solutions are posted on 15 January at 17.00 on

abelkonkurransen.no