The Niels Henrik Abel mathematics competition: First round 2018–2019

8 November 2018 (English)

ABEL KONKURRANSEN

Do not turn the page until told to by your teacher!

The first round of the Abel competition consists of 20 multiple choice problems to be solved in 100 minutes. Only one of the five alternatives is correct. Write your answers in the lower left hand side of the form.

You get 5 points for each correct answer, 1 point for a blank answer, and 0 points for a wrong answer. This yields a total between 0 and 100 points. A totally blank response results in 20 points.

No aids other than scratch paper and writing implements (including compass and ruler, but not protactor) are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name	Date of birth
Address	Gender F ☐ M ☐
Post code	Post office
School	Class
☐ Check the box to allow us to put your name on the results list. (Regardless, we only publish results for the best third.)	

Answers

1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

For the teacher

Correct: ☐ $\cdot\, 5 =$ ☐

Unanswered: $+$ ☐

Points: $=$ ☐

Problem 1

In a restaurant one can choose between four different appetisers, five different main courses, and five different desserts. How many different three course dinners can one order here?

A 3 B 14 C 29 D 100 E 125

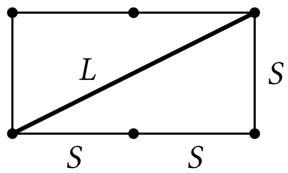
Problem 2

If $\tau = 2\pi$, which one of these expressions equals the area of a circle of radius 1?

A τ B $\frac{\tau}{2}$ C τ^2 D $\frac{\tau^2}{4}$ E 2τ

Problem 3

A construction set contains short (S) and long (L) rods and a way to connect them together. First you make a rectangle where one side consists of one short rod and the other side consists of two short rods. You note that the long rod fits exactly along the diagonal. Then you make a rectangle where one side consists of one long rod and the other side consists of two long rods. How many short rods will now fit along the diagonal?



A 4 B 5 C 6 D 7 E not a whole number

Problem 4

The fraction $\dfrac{2+\sqrt{2}}{1+\sqrt{2}}$ can be simplified to

A $\frac{4}{3}$ B $\frac{3}{2}$ C $\sqrt{2}$ D $1+\sqrt{2}$ E none of these

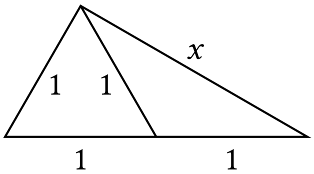
Problem 5

The equation $2^{x+1} + 2^{x-4} = 5^2 + 2^{x-1}$ has the solution:

A $x = \frac{1}{2}$ B $x = 1$ C $x = 2$ D $x = 3$ E $x = 4$

Problem 6

What is the value of x in the diagram?



A 1 B $\sqrt{2}$ C $\sqrt{3}$ D 2 E $\sqrt{5}$

Problem 7

Which one of the following numbers is not a prime?

A 2^4+1 B 2^8+1 C 100^2-99^2 D $2\cdot3\cdot5\cdot7+1$ E $2\cdot3\cdot5\cdot7-1$

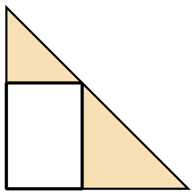
Problem 8

In how many ways can 210 be written as the product of three different positive integers? We do not account for order, so $3\cdot7\cdot10$ and $10\cdot3\cdot7$ are not considered different.

A 6 B 9 C 10 D 13 E 19

Problem 9

A right isosceles triangle has area 72. A rectangle is inscribed with two sides along the legs of the triangle and one corner on the hypotenuse. The two small triangles have combined area 40. How long is the shortest side of the rectangle?



A 3 B 4 C 5 D 6 E none of these

Problem 10

A shelf in front of you holds a row of glasses. You put one pea in the first glass. In the next glass you put three peas, so that glass number two contains two more peas than the first glass. In the third glass you put four more peas than in glass number two, and in the fourth glass you put eight more peas than in glass number three. You continue in this way, doubling the difference in the number of peas from one glass to the next. The number of peas you must put in glass number n is then

A $2n - 1$ B $n^2 - n + 1$ C $3^{n-1} - (n-1)(n-2)2^{n-3}$ D $2^{n-1} + 2^{n-2}$ E $2^n - 1$

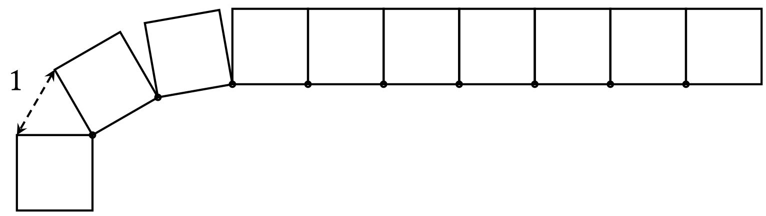
Problem 11

A school class consists of six boys and seven girls. In how many ways can you put a group together, if it is to have five members including at least one member of each gender?

A 1245 B 1260 C 1276 D 4096 E 6930

Problem 12

Ten squares with side length 1 are connected in a row by their lower neighbouring corners. We rotate the squares so that the upper neighbouring corners are moved a distance 1 apart.



Before we have rotated all the squares, we come all the way round so that the next square will overlap the first. We remove this square and all the ones after it. How many squares are left?

A 5 B 6 C 7 D 8 E 9

Problem 13

In the country Financia they use three types of coins. These have values 7 kr, 8 kr, and 9 kr. Even if one has access to an unlimited number of coins, there are some sums one cannot make, such as 11 kr. What is the largest integer sum that one *cannot* make using a combination of such coins?

A 13 kr B 19 kr C 20 kr D 25 kr E greater than 25 kr

Problem 14

In the first round of the Abel competition there are twenty problems. For each problem one gets five points for a correct answer, one point for not answering, and zero points for a wrong answer. All the pupils at Lurholmen high school participate in the competition, and no two of them get the same total score. What is the greatest possible number of pupils at Lurholmen high school?

A 80 B 94 C 95 D 97 E 100

Problem 15

In how many ways can we cover a 3×10 grid using ten identical rectangular pieces of size 3×1?

A 28 B 30 C 89 D 120 E 243

Problem 16

How many real numbers x between $1/5$ and 5 have the property that $x + 1/x$ and $x^2 + 1/x^2$ are both integers?

A 10 B 9 C 8 D 7 E 6

Problem 17

How many times does the factor 2 appear in the prime factorisation of the number $2018! = 1 \cdot 2 \cdot \cdots \cdot 2018$?

A 2000 B 2004 C 2007 D 2011 E 2018

Problem 18

For how many integers $n \geq 1$ is the expression $3^n - n^2$ a prime?

A 1 B 2 C 3 D 4 E more than 4

Problem 19

The constants a, b, c, and d are all nonzero. Two first degree functions f and g are given by the equations $f(x) = ax + b$ and $g(x) = cx + d$. If you are told that $f(d) = g(b) = 0$, what can you say with certainty about f and g?

- A f and g are the same function.
- B The graphs of f and g are parallel lines.
- C The graphs of f and g intersect at a right angle.
- D The mirror image of the graph of f through the line $y = x$ is the graph of g.
- E None of the above statements is necessarily true.

Problem 20

In a particular country all the major cities have an airport, but each airport has a direct connection with at most three other major cities. All connections go both ways. Further, one can travel between any two major cities with at most one plane change. What is the largest possible number of major cities in the country?

A 8 B 9 C 10 D 12 E 13

The solutions are published on 9 November at 17:00 on

abelkonkurransen.no