The Niels Henrik Abel mathematics competition: Second round 2018–2019

10 January 2019 (English)

ABEL KONKURRANSEN

Do not turn the page until told to by your teacher!

The second round of the Abel competition consists of 10 problems to be solved in 100 minutes. The solutions are integers between 0 and 999, inclusive. Write your answers in the lower left hand side of the form.

You get 10 points for each correct answer and 0 points for a blank or wrong answer. This yields a total between 0 and 100 points.

No aids other than scratch paper and writing implements (including compass and ruler, but not protactor) are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name			Date of birth
			Gender F ☐ M ☐
School			Class
Citizenship	Email		Mobile phone

☐ Check the box if you participated in round 1 **online**.

☐ Check the box to allow us to put your name on the score board. (Only applies to the highest scores, approx. top 33 %.)

Answers

1		6	
2		7	
3		8	
4		9	
5		10	

For the teacher

Correct: ☐ · 10 = ☐

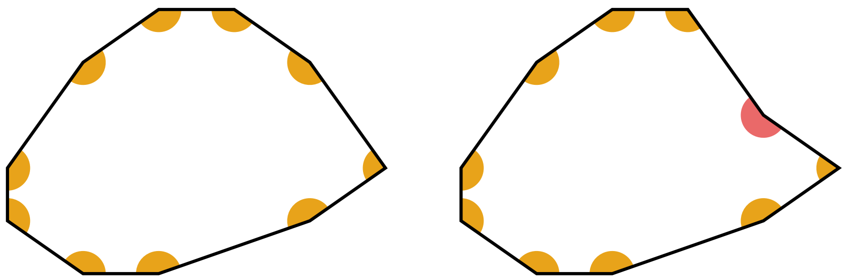
Problem 1

Roger Rabbit jumps up a staircase which has 12 steps. He jumps either one or two steps at a time, and never reverses direction. In how many ways can he jump all the way to the top of the staircase?

Problem 2

Every angle in a convex decagon (ten-sided polygon) $ABCDEFGHIJ$ is measured in degrees, and every measure is an integer. What is the smallest possible value of $A + B + C + D + E$?

The left decagon in the figure is convex. The one to the right is not convex.



Problem 3

The quadratic polynomial P satisfies $P(1) = 1$, $P(2) = 8$ and $P(3) = 27$. What is the value of $P(-9)$?

Problem 4

Nils writes the 90-digit integer 987654321…987654321 on a blackboard, that is the digits 987654321 ten times in succession. Then he erases two of the digits. The 88-digit number he ends up with is divisible by 9. How many different possible numbers could he have ended up with?

Problem 5

The triangular numbers $T_1, T_2, T_3, \ldots$ are defined by

$$T_n = \frac{n(n+1)}{2}.$$

What is the sum of all positive integers n for which T_{n+4} is divisible by T_n?

Problem 6

How many of the 999 positive integers less than 1000 can be written as $a^2 - b^2$, where a and b are integers?

Problem 7

Positive integers a, b, c satisfy the equations $2a + 3b = 5c$ and $a + b + c = 2019$. What is the largest possible value of c?

Problem 8

How many consecutive zeros come immediately after the decimal point in the decimal representation of $10^{320} - \sqrt{10^{640} - 1}$?

Problem 9

What is the largest natural number less than 1000 which is divisible by exactly 20 natural numbers, including 1 and the number itself?

Problem 10

The surface of a convex polyhedron consists of 30 squares, 20 regular hexagons, and 12 regular decagons (ten-sided polygons). How many vertices (corners) does it have?

Solutions are posted on 11 January at 17.00 on
abelkonkurransen.no