

The Niels Henrik Abel mathematics
competition: Final 2019–2020

3 March 2020 (English)



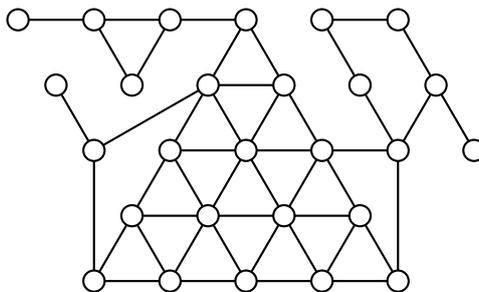
In the final round of the Abel contest there are four problems (seven sub-problems) to be solved in four hours. You are required to justify your answers. **Start a new sheet of paper for each of the four problems.**

You can score up to 10 points for each problem. The maximum score is thus 40.

No aids other than writing paper, writing tools and bilingual dictionaries are permitted.

Problem 1

a. In how many ways can the circles be coloured using three colours, so that no two circles connected by a line segment have the same colour?



b. A round table has room for n diners ($n \geq 2$). There are napkins in three different colours. In how many ways can the napkins be placed, one for each seat, so that no two neighbours get napkins of the same colour?

Problem 2

a. Find all natural numbers k such that there exist natural numbers a_1, a_2, \dots, a_{k+1} with

$$a_1! + a_2! + \dots + a_{k+1}! = k!.$$

Note that we do not consider 0 to be a natural number.

b. Assume that a and b are natural numbers with $a \geq b$ so that $\sqrt{a + \sqrt{a^2 - b^2}}$ is a natural number. Show that a and b have the same parity.

Problem 3

Show that the equation

$$x^2 \cdot (x - 1)^2 \cdot (x - 2)^2 \cdot \dots \cdot (x - 1008)^2 \cdot (x - 1009)^2 = c$$

has 2020 real solutions, provided $0 < c < \frac{(1009 \cdot 1007 \cdot \dots \cdot 3 \cdot 1)^4}{2^{2020}}$.



Problem 4

- a. The midpoint of the side AB in the triangle ABC is called C' . A point on the side BC is called D , and E is the point of intersection of AD and CC' . Assume that $AE/ED = 2$. Show that D is the midpoint of BC .
- b. The triangle ABC has a right angle at A . The centre of the circumcircle is called O , and the base point of the normal from O to AC is called D . The point E lies on AO with $AE = AD$. The angle bisector of $\angle CAO$ meets CE in Q . The lines BE and OQ intersect in F . Show that the lines CF and OE are parallel.