The Niels Henrik Abel Contest 1993

Problem 1

If $1 - \dfrac{1}{1-x} = \dfrac{1}{1-x}$, then x equals

A) -2 B) -1 C) $\dfrac{1}{2}$ D) 2 E) 3

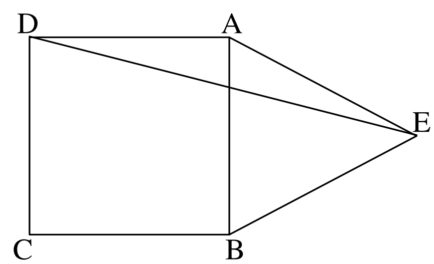
Problem 2

A square is changed into a ractange by increasing to of its sides by $p\%$ and reducing the two others by $p\%$. The area is then reduced by 1%. The value of p is

A) $\dfrac{1}{2}$ B) 1 C) 5 D) 10 E) 11

Problem 3

On the figure, $ABCD$ is a square and ABE is an equilateral triangle. Then, $\angle AED$ equals



A) $10°$ B) $12\frac{1}{2}°$ C) $15°$

D) $20°$ E) $22\frac{1}{2}°$

Problem 4

The greatest number of $\sqrt{2}$, $\sqrt[3]{3}$, $3-\sqrt{6}$, and $1+\frac{1}{\pi}$ is

A) $\sqrt{2}$ B) $\sqrt[3]{3}$ C) $3-\sqrt{6}$ D) $1+\dfrac{1}{\pi}$
E) Two of them are greatest

Problem 5

The number of pairs of real numbers (x, y) satisfying the equations $x = x^2 + y^2$ and $y = 2xy$, is

A) 0 B) 1 C) 2 D) 3 E) 4

Problem 6

If $2x - y = 1$, $2y - z = 2$, and $2z - x = 3$, then $x + y + z$ equals

A) 1 B) 2 C) 3 D) 4 E) None of these

Problem 7

If $(3x-1)^7 = a_7x^7 + a_6x^6 + \cdots + a_1x + a_0$ fr all x, then $a_0 + a_1 + \cdots + a_6 + a_7$ equals

A) 7 B) 10 C) 64 D) -64 E) 128

Problem 8

The fraction $\frac{2(\sqrt{2}+\sqrt{6})}{3\left(\sqrt{2+\sqrt{3}}\right)}$ equals

A) $\dfrac{2\sqrt{2}}{3}$ B) 1 C) $\dfrac{2\sqrt{3}}{3}$ D) $\dfrac{4}{3}$ E) $\dfrac{16}{9}$

Problem 9

If $g(x) = 1 - x^2$ og $f(g(x)) = \frac{1-x^2}{x^2}$ for $x \neq 0$, then $f\left(\frac{1}{2}\right)$ equals

A) $\dfrac{3}{4}$ B) 1 C) $\sqrt{2}$ D) 3 E) $\dfrac{\sqrt{2}}{2}$

Problem 10

The sum of the real solutions to the equation $x^2 + x + 1 = \frac{156}{x^2+x}$ is

A) 13 B) 6 C) -1 D) -2 E) None of these

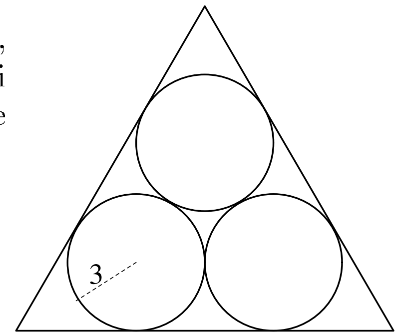
Problem 11

The least integer value of k so that the equation $x(k-x) = 4$ has no real solution, is

A) -5 B) -4 C) -3 D) 3 E) 0

Problem 12

In the figure, the three equally large circles touch, and they touch the edges of the triangle. If the radii of the circles are 3, the circumference of the triangle is



A) $36+9\sqrt{2}$ B) $36+6\sqrt{3}$ C) $36+9\sqrt{3}$
D) $18+18\sqrt{3}$ E) 45

Problem 13

Five animals — A, B, C, D, and E — are either wolves or dogs. Dogs always tell the truth, whereas wolves always lie. A claims that B is a dog. C claims that D is a wolf. E claims that A is a dog. B claims that C is a wolf. D claims that B and E are different kinds. The number of wolves is

A) 1 B) 2 C) 3 D) 4 E) 5

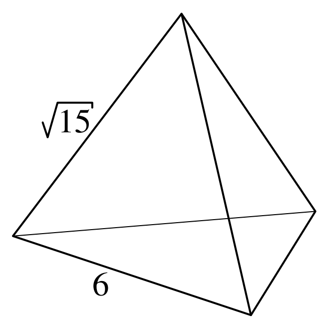
Problem 14

If you add 1 litre of water to a solution consisting of acid and water, the new solution will consist of 20% acid. If you add another 1 litre of acid to the new solution, it will contain $33\frac{1}{3}\%$ acid. The concentration of acid in the original solution was

A) 20% B) 22,5% C) 24% D) 25% E) $26\dfrac{2}{3}\%$

Problem 15

An equilateral triangle with sides of length 6 is the bottom of a pyramide with edges of length $\sqrt{15}$. Then, the volume of the pyramide is



A) 9 B) 10 C) $\dfrac{9}{2}\sqrt{3}$ D) $\dfrac{9}{2}\sqrt{5}$
E) None of these

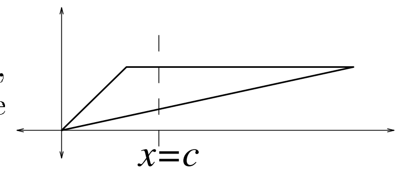
Problem 16

For a and n natural numbers, let k be the greatest natural number such that $n > ka$. If we define $n \otimes a = n(n-a)(n-2a)\cdots(n-ka)$, then $\frac{72\otimes 8}{18\otimes 2}$ equals

A) 4^5 B) 4^7 C) 4^8 D) 4^9 E) None of these

Problem 17

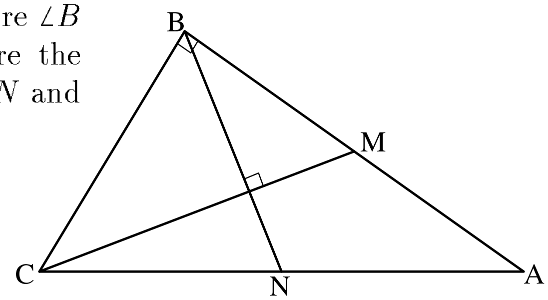
The line $x = c$ cuts the triangle with corners $(0,0)$, $(1,1)$, and $(9,1)$ into two regions. For the area of the two regions to be the same, c must be



A) $\frac{5}{2}$ B) 3 C) $\frac{7}{2}$ D) $2\sqrt{3}$ E) $\sqrt{10}$

Problem 18

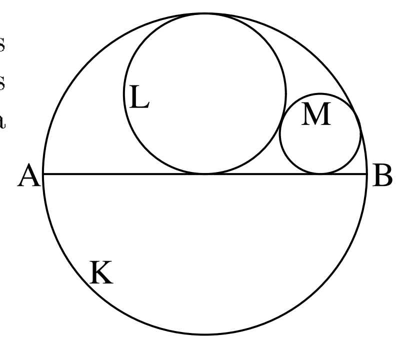
In the straight angled triangle ABC where $\angle B$ is straight and $BC = 1$, M and N are the midpoints of AB and AC. If the lines BN and CM are perpendicular, then CM equals



A) $\sqrt{2}$ B) $\frac{3}{2}\sqrt{2}$ C) $2\sqrt{2}$
D) $\frac{1}{2}\sqrt{5}$ E) $\frac{1}{2}\sqrt{6}$

Problem 19

AB is a dimaiter of the circle K. The circle L touches K, and AB in the centre of K. The circle M touches K, L, and AB. Then, the quotient between the area of K and the area of M is



A) 12 B) 14 C) 16 D) 18
E) None of these

Problem 20

If a, b, and c are three positive integers such that $abc+ab+ac+bc+a+b+c = 1000$, then $a + b + c$ equals

A) 28 B) 43 C) 36 D) 42 E) 24