The Niels Henrik Abel Contest 1995–96
FINAL

Problem 1

Let S be a circle with centre C and radius r, and let P be an arbitrary point other than C. Draw a line l through P which intersects the circle in two points X and Y. Let Z be the midpoint of XY. Prove that the points Z for the various lines l lies on a circle. Find the centre and radius of this circle.

Problem 2

For $n \in \mathbf{N}$, prove that $\left\lfloor \sqrt{n} + \sqrt{n+1} \right\rfloor = \left\lfloor \sqrt{4n+1} \right\rfloor$. (Here, $\lfloor x \rfloor$ denotes x rounded downwards to nearest integer: eg. $\lfloor 1.345 \rfloor = 1$, $\lfloor 3.864 \rfloor = 3$, and $\lfloor 5 \rfloor = 5$.)

Problem 3

Per and Kari each have n pieces of paper. They both write down the numbers from 1 to $2n$ in an arbitrary sequence: one number on each side. Afterwards, they are to place the pieces of paper on a table with one side showing. Prove that they may always place them so as to make all the numbers from 1 to $2n$ visible at once.

Problem 4

Let $f : \mathbf{N} \to \mathbf{N}$ (ie. both x and $f(x)$ natural numbers) be such that $f(f(1995)) = 95$, $f(xy) = f(x)f(y)$, and $f(x) \leq x$. Find all possible values of $f(1995)$.