The Niels Henrik Abel Contest 1996–97
Second round

Problem 1

On a cube, 27 points are marked in the following manner: one point in each corner, one point on the middle of each edge, one point on the middle of each face, and one in the middle of the cube. The number of lines containing three out of these points is

A) 33 B) 42 C) 49 D) 72 E) 81

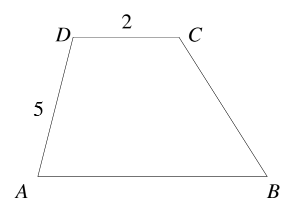
Problem 2

Let x, y, z natural numbers so that $xyz = 78$ and $x^2 + y^2 + z^2 = 206$. What is $x + y + z$?

A) 18 B) 20 C) 30 D) 42 E) None of these

Problem 3

Let $ABCD$ be a trapezoid with AB and CD parallel, $\angle D = 2\angle B$, $AD = 5$, and $CD = 2$. Then, AB equals



A) 7 B) 8 C) $\frac{13}{2}$ D) $\frac{27}{4}$ E) $5 + \frac{3\sqrt{2}}{2}$

Problem 4

Three friends are to divide fice different jobs between each other so that nobody is left without a job. In how many different ways can this be done?

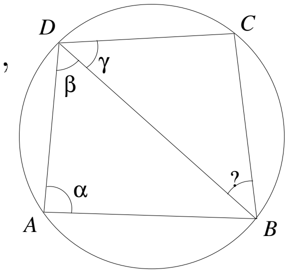
A) 6 B) 25 C) 40 D) 90 E) 150

Problem 5

Let f be a function from the non-negative integers to the non-negative integers such that $f(nm) = nf(m) + mf(n)$, $f(10) = 19$, $f(12) = 52$, and $f(15) = 26$. What is $f(8)$?

A) 12 B) 24 C) 36 D) 48 E) 60

Problem 6



A square $ABCD$ is inscribed in a circle. Let $\alpha = \angle DAB$, $\beta = \angle BDA$, and $\gamma = \angle CDB$. Then, $\angle DBC$ equals

A) $\alpha - \beta$ B) $\alpha - \gamma$ C) $90^\circ - \alpha + \beta$
D) $90^\circ - \alpha + \gamma$ E) $180^\circ - \alpha - \gamma$

Problem 7

If 1, 2, and 3 are solutions to the equation $x^4 + ax^2 + bx + c = 0$, så then $a + c$ equals

A) -12 B) 24 C) 35 D) -61 E) -63

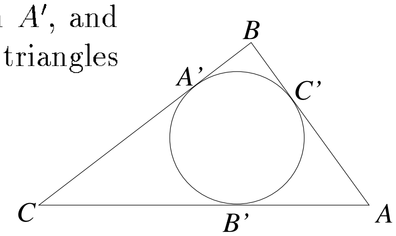
Problem 8

Let x and y be positive integers. The least possible value of $|11x^5 - 7y^3|$ is

A) 1 B) 2 C) 3 D) 4 E) None of these

Problem 9

The triangle ABC has vertices $AB = 3$, $BC = 4$, $AC = 5$. The inscribed sircle is tangent to AB in C', BC in A', and AC in B'. What is the ratio between the areas of the triangles $A'B'C'$ and ABC?



A) $\frac{1}{4}$ B) $\frac{1}{5}$ C) $\frac{2}{9}$ D) $\frac{4}{21}$ E) $\frac{5}{24}$

Problem 10

Let $x_1, x_2, \ldots, x_5$ be non-negative real numbers such that $x_1 + x_2 + \cdots + x_5 = 100$. Let M be the maximum of the numbers $x_1 + x_2$, $x_2 + x_3$, $x_3 + x_4$ og $x_4 + x_5$. The least possible value of M lies in the interval

A) $[0, 32\rangle$ B) $[32, 34\rangle$ C) $[34, 36\rangle$ D) $[36, 38\rangle$ E) $[38, 40]$