The Niels Henrik Abel Contest 1997–98
Second round

Problem 1

In a class, some puppils learn German, the other learn French. The number of girls learning French and the number of boys learning German total to 16. There are 11 pupils learning French, and there are 10 girls in the class. In addition to the girls learning French, there are 16 pupils. How many pupils are there in the class?

A) 18 B) 21 C) 23 D) 27 E) 31

Problem 2

How many real solutions are there to the equation $|x - |2x + 1|| = 3$. (Here, $|x|$ denotes the absolute value of x: ie., if $x \geq 0$, then $|x| = |-x| = x$.)

A) 0 B) 1 C) 2 D) 3 E) 4

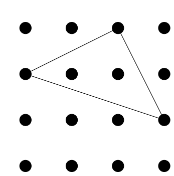
Problem 3

Let $f_i(x)$, $i = 1, 2, 3, \ldots$ be defined by $f_1(x) = \frac{1}{1-x}$ and $f_{i+1}(x) = f_i(f_1(x))$. Then, $f_{1998}(1998)$ equals

A) 0 B) 1998 C) $-\frac{1}{1997}$ D) $\frac{1997}{1998}$ E) None of these

Problem 4

We have a square grid of 4 times 4 points. How many triangles are there with vertices on the points? (The three vertices may not lie on a line.)



A) 252 B) 256 C) 360 D) 516 E) 560

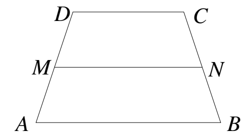
Problem 5

Determine $m > 0$ so that $x^4 - (3m+2)x^2 + m^2 = 0$ has four real solutions forming an arithmetic series: ie., that the solutions may be written a, $a + b$, $a + 2b$, and $a + 3b$ for suitable a and b.

A) 1 B) 3 C) 7 D) 12 E) None of these

Problem 6

Let $ABCD$ be a trapezoid with $AB||CD$. Let $a = AB$ and $b = CD$. For $MN||AB$ such that M lies on AD, N lies on BC, and the trapezoids $ABNM$ and $MNCD$ have the same area, the length MN equals



A) $\sqrt{ab}$ B) $\dfrac{a+b}{2}$ C) $\dfrac{a^2+b^2}{a+b}$ D) $\sqrt{\dfrac{a^2+b^2}{2}}$
E) $\dfrac{a^2+(2\sqrt{2}-2)ab+b^2}{\sqrt{2}(a+b)}$

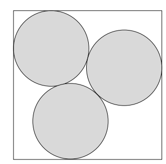
Problem 7

For how many integer values of m does the lines $13x+11y=700$ and $y=mx-1$ intersect in a point with integer valued coordinates?

A) None B) 1 C) 2 D) 3 E) Infinitely many

Problem 8

Place three discs with radius r in a square with sides of length 1 so that the discs do not intersect: as on the figure. What is the greatest possible value of r?



A) $\dfrac{1}{3}$ B) $\dfrac{1}{4}$ C) $\dfrac{\sqrt{2}}{6}$ D) $2\sqrt{2}-\sqrt{6}$ E) $\dfrac{\sqrt{2}}{1+2\sqrt{2}+\sqrt{3}}$

Problem 9

What is the sum $\dfrac{1}{1\cdot 2\cdot 3}+\dfrac{1}{2\cdot 3\cdot 4}+\cdots+\dfrac{1}{1996\cdot 1997\cdot 1998}$ lik?

A) $\dfrac{2\cdot 1997}{3\cdot 1996\cdot 1998}$ B) $\dfrac{1}{3}-\dfrac{1}{3\cdot 1998}$ C) $\dfrac{1}{4}-\dfrac{1}{1997^2}$
D) $\dfrac{1}{3}-\dfrac{1}{3\cdot 1997\cdot 1998}$ E) $\dfrac{1}{4}-\dfrac{1}{2\cdot 1997\cdot 1998}$

Problem 10

The minimal value of $f(x)=\sqrt{a^2+x^2}+\sqrt{(x-b)^2+c^2}$ is

A) $a+b+c$ B) $\sqrt{a^2+(b+c)^2}$ C) $\sqrt{b^2+(a+c)^2}$
D) $\sqrt{(a+b)^2+c^2}$ E) None of these