41st IMO 2000

A1. AB is tangent to the circles $CAMN$ and $NMBD$. M lies between C and D on the line CD, and CD is parallel to AB. The chords NA and CM meet at P; the chords NB and MD meet at Q. The rays CA and DB meet at E. Prove that $PE = QE$.

A2. A, B, C are positive reals with product 1. Prove that $(A - 1 + \frac{1}{B})(B - 1 + \frac{1}{C})(C - 1 + \frac{1}{A}) \leq 1$.

A3. k is a positive real. N is an integer greater than 1. N points are placed on a line, not all coincident. A *move* is carried out as follows. Pick any two points A and B which are not coincident. Suppose that A lies to the right of B. Replace B by another point B' to the right of A such that $AB' = kBA$. For what values of k can we move the points arbitrarily far to the right by repeated moves?

B1. 100 cards are numbered 1 to 100 (each card different) and placed in 3 boxes (at least one card in each box). How many ways can this be done so that if two boxes are selected and a card is taken from each, then the knowledge of their sum alone is always sufficient to identify the third box?

B2. Can we find N divisible by just 2000 different primes, so that N divides $2^N + 1$? [N may be divisible by a prime power.]

B3. $A_1A_2A_3$ is an acute-angled triangle. The foot of the altitude from A_i is K_i and the incircle touches the side opposite A_i at L_i. The line K_1K_2 is reflected in the line L_1L_2. Similarly, the line K_2K_3 is reflected in L_2L_3 and K_3K_1 is reflected in L_3L_1. Show that the three new lines form a triangle with vertices on the incircle.

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