**42nd IMO 2001**

**A1**. $ABC$ is acute-angled. $O$ is its circumcenter. $X$ is the foot of the perpendicular from $A$ to $BC$. $\angle C \geq \angle B + 30^o$. Prove that $\angle A + \angle COX < 90^o$.

**A2**. $a, b, c$ are positive reals. Prove that $\frac{a}{\sqrt{a^2+8bc}} + \frac{b}{\sqrt{b^2+8ca}} + \frac{c}{\sqrt{c^2+8ab}} \geq 1$.

**A3**. Integers are placed in each of the 441 cells of a $21 \times 21$ array. Each row and each column has at most 6 different integers in it. Prove that some integer is in at least 3 rows and at least 3 columns.

**B1**. Let $n_1, n_2, \ldots, n_m$ be integers where $m$ is odd. Let $x = (x_1, \ldots, x_m)$ denote a permutation of the integers $1, 2, \ldots, m$. Let $f(x) = x_1 n_1 + x_2 n_2 + \cdots + x_m n_m$. Show that for some distinct permutations $a, b$ the difference $f(a) - f(b)$ is a multiple of $m!$.

**B2**. $ABC$ is a triangle. $X$ lies on $BC$ and $AX$ bisects $\angle A$. $Y$ lies on $CA$ and $BY$ bisects $\angle B$. $\angle A = 60^o$. $AB + BX = AY + YB$. Find all possible values for $\angle B$.

**B3**. $K > L > M > N$ are positive integers such that $KM + LN = (K + L - M + N)(-K + L + M + N)$. Prove that $KL + MN$ is composite.


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