40th IMO 1999

A1. Find all finite sets S of at least three points in the plane such that for all distinct points A, B in S, the perpendicular bisector of AB is an axis of symmetry for S.

A2. Let $n \geq 2$ be a fixed integer. Find the smallest constant C such that for all non-negative reals $x_1, \ldots, x_n$:

$$\sum_{i<j} x_i x_j (x_i^2 + x_j^2) \leq C(\sum_i x_i)^4.$$

Determine when equality occurs.

A3. Given an $n \times n$ square board, with n even. Two distinct squares of the board are said to be adjacent if they share a common side, but a square is not adjacent to itself. Find the minimum number of squares that can be marked so that every square (marked or not) is adjacent to at least one marked square.

B1. Find all pairs (n, p) of positive integers, such that: p is prime; $n \leq 2p$; and $(p-1)^n + 1$ is divisible by n^{p-1}.

B2. The circles C_1 and C_2 lie inside the circle C, and are tangent to it at M and N, respectively. C_1 passes through the center of C_2. The common chord of C_1 and C_2, when extended, meets C at A and B. The lines MA and MB meet C_1 again at E and F. Prove that the line EF is tangent to C_2.

B3. Determine all functions $f : R \to R$ such that $f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1$ for all x, y in R. [R is the reals.]

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