

15-th Nordic Mathematical Contest

March 29, 2001

1. Let A be a finite set of unit squares in the coordinate plane, each of which has vertices at integer points. Show that there exists a subset B of A consisting of at least $1/4$ of the squares in A such that no two distinct squares in B have a common vertex.

2. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is bounded and satisfies

$$f\left(x + \frac{1}{3}\right) + f\left(x + \frac{1}{2}\right) = f(x) + f\left(x + \frac{5}{6}\right)$$

for all real x . Show that f is periodic.

3. Find the number of real roots of the equation

$$x^8 - x^7 + 2x^6 - 2x^5 + 3x^4 - 3x^3 + 4x^2 - 4x + \frac{5}{2} = 0.$$

4. Each of the diagonals AD , BE and CF of a convex hexagon $ABCDEF$ divides its area into two equal parts. Prove that these three diagonals pass through the same point.