

19-th Nordic Mathematical Contest

April 5, 2005

1. Find all positive integers k such that the product of the decimal digits of k equals $\frac{25}{8}k - 211$.
2. If a, b, c are positive numbers, prove the inequality

$$\frac{2a^2}{b+c} + \frac{2b^2}{c+a} + \frac{2c^2}{a+b} \geq a+b+c.$$

3. There are 2005 boys and girls sitting at a round table. No more than 668 of them are boys. A girl G is said to be in a *strong position* if, counting from G to either direction at any length (G herself included), the number of girls is always strictly larger than the number of boys. Prove that there always exists a girl in a strong position.
4. Circle \mathcal{C}_1 touches circle \mathcal{C}_2 internally at A . A line through A intersects \mathcal{C}_1 again at B and \mathcal{C}_2 again at C . The tangent to \mathcal{C}_1 at B intersects \mathcal{C}_2 at D and E . The tangents to \mathcal{C}_1 through C touch \mathcal{C}_1 at F and G . Prove that points D, E, F, G are concyclic.