

20th Nordic Mathematical Contest

Thursday March 30, 2006

English version

Time allowed: 4 hours. Each problem is worth 5 points.

Problem 1. Let B and C be points on two fixed rays emanating from a point A such that $AB + AC$ is constant.

Prove that there exists a point $D \neq A$ such that the circumcircles of the triangles ABC pass through D for every choice of B and C .

Problem 2. The real numbers x , y and z are not all equal and fulfil

$$x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x} = k$$

Determine all possible values of k .

Problem 3. A sequence of positive integers $\{a_n\}$ is given by

$$a_0 = m \quad \text{and} \quad a_{n+1} = a_n^5 + 487 \quad \text{for all } n \geq 0$$

Determine all values of m for which the sequence contains as many square numbers as possible.

Problem 4. The squares of a 100×100 chessboard are painted with 100 different colours. Each square has only one colour and every colour is used exactly 100 times.

Show that there exists a row or a column on the chessboard in which at least 10 colours are used.

Only writing and drawing sets are allowed