

The 28th Nordic Mathematical Contest

Monday, 31 March 2014

English Version

*The time allowed is 4 hours. Each problem is worth 5 points.
The only permitted aids are writing and drawing tools.*

Problem 1

Find all functions $f : \mathbb{N} \rightarrow \mathbb{N}$ (where \mathbb{N} is the set of the natural numbers and is assumed to contain 0), such that

$$f(x^2) - f(y^2) = f(x+y)f(x-y),$$

for all $x, y \in \mathbb{N}$ with $x \geq y$.

Problem 2

Given an equilateral triangle, find all points inside the triangle such that the distance from the point to one of the sides is equal to the geometric mean of the distances from the point to the other two sides of the triangle.

[The geometric mean of two numbers x and y equals \sqrt{xy} .]

Problem 3

Find all nonnegative integers a, b, c , such that

$$\sqrt{a} + \sqrt{b} + \sqrt{c} = \sqrt{2014}.$$

Problem 4

A game is played on an $n \times n$ chessboard. At the beginning there are 99 stones on each square. Two players A and B take turns, where in each turn the player chooses either a row or a column and removes one stone from each square in the chosen row or column. They are only allowed to choose a row or a column, if it has least one stone on each square. The first player who cannot move, loses the game. Player A takes the first turn. Determine all n for which player A has a winning strategy.