# The 32nd Nordic Mathematical Contest

## Monday, 9 April 2018

## English version

*Time allowed: 4 hours. Each problem is worth 7 points.*
*Only writing and drawing tools are allowed.*

**Problem 1** Let $k$ be a positive integer and $P$ a point in the plane. We wish to draw lines, none passing through $P$, in such a way that any ray starting from $P$ intersects at least $k$ of these lines. Determine the smallest number of lines needed.

**Problem 2** A sequence of primes $p_1, p_2, \ldots$ is given by two initial primes $p_1$ and $p_2$, and $p_{n+2}$ being the greatest prime divisor of $p_n + p_{n+1} + 2018$ for all $n \geq 1$. Prove that the sequence only contains finitely many primes for all possible values of $p_1$ and $p_2$.

**Problem 3** Let $ABC$ be a triangle with $AB < AC$. Let $D$ and $E$ be on the lines $CA$ and $BA$, respectively, such that $CD = AB$, $BE = AC$, and $A$, $D$ and $E$ lie on the same side of $BC$. Let $I$ be the incentre of triangle $ABC$, and let $H$ be the orthocentre of triangle $BCI$. Show that $D$, $E$, and $H$ are collinear.

**Problem 4** Let $f = f(x, y, z)$ be a polynomial in three variables $x, y, z$ such that

$$f(w, w, w) = 0$$

for all $w \in \mathbb{R}$. Show that there exist three polynomials $A, B, C$ in these same three variables such that $A + B + C = 0$ and

$$f(x, y, z) = A(x, y, z) \cdot (x - y) + B(x, y, z) \cdot (y - z) + C(x, y, z) \cdot (z - x).$$

Is there any polynomial $f$ for which these $A, B, C$ are uniquely determined?