6-th Nordic Mathematical Contest

April 8, 1992

1. Find all real numbers $x, y, z > 1$ that satisfy the equality
$$x + y + z + \frac{3}{x-1} + \frac{3}{y-1} + \frac{3}{z-1} = 2\left(\sqrt{x+2} + \sqrt{y+2} + \sqrt{z+2}\right).$$

2. Let $a_1, a_2, \ldots, a_n$ $(n \geq 2)$ be distinct positive integers. Prove that the polynomial $f(x) = (x-a_1)(x-a_2)\cdots(x-a_n) - 1$ is irreducible over $\mathbb{Z}[x]$.

3. Prove that among all triangles with the inradius 1, the one with the smallest perimeter is the equilateral triangle.

4. Peter has many black and white unit squares and wants to construct a $n \times n$ square using them in such a way that no four vertices of a rectangle with sides parallel to those of the big square are of the same color. How big can n be?

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